On the terminal costs in LQ games and the application to receding horizon games

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Game theory meets MPC: advances in multi-agent control

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SYSTEMS CONTROL AND MULTIAGENT OPTIMIZATION RESEARCH

1. Introduction
2. From finite to infinite-horizon
3. Implications for receding horizon games
4. Open problems and future directions

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Infinite-horizon control problem

$$\begin{aligned} \min \quad & J\left(x_0, \mathbf{u}\right) \\ \text{s.t.} \quad & x_{t+1} = f_t\left(x_t, u_t\right) \\ & g_t\left(x_t, u_t\right) \end{aligned}$$

The goals are:

1. Stabilize the system
2. Minimize the cost function
3. Satisfy the constraints

How does receding horizon perform?

	Receding horizon control
Stability	1. Adding a terminal cost 2. Extending the horizon
Optimality	Approximation of the original solution

Can we extend to receding horizon games?

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Linear quadratic games

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Which Nash equilibrium is approximated when using receding horizon? Can we choose?

Main results

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2. Terminal costs act as an equilibrium selector.

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Salizzoni, G., Hall, S., & Kamgarpour, M. (2025)
Bridging Finite and Infinite-Horizon Nash Equilibria in Linear Quadratic Games.
arXiv preprint arXiv:2508.20675.

Single agent - Optimal solution for the finite-horizon LQR

$$P_t = Q + A^\top P_{t+1} A - A^\top P_{t+1} B (R + B^\top P_{t+1} B)^{-1} B^\top P_{t+1} A, \quad P_T = Q_T$$

$$K_t^* = -(R + B^\top P_t B)^{-1} B^\top P_t A$$

→

$$P_t = f(P_{t+1})$$

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Multi agent - Nash equilibrium for the finite-horizon LQR

$$\left(P_t^1, P_t^2, \ldots, P_t^N\right) = f\left(P_{t+1}^1, P_{t+1}^2, \ldots, P_{t+1}^N\right) \qquad P_T^i = Q_T^i \ \forall i \in [1, N]$$

$$\left(K_t^1, K_t^2, \ldots, K_t^N\right) = g\left(P_t^1, P_t^2, \ldots, P_t^N\right)$$

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$$\mathbf{P}_t = f\left(\mathbf{P}_{t+1}\right), \quad \mathbf{P}_T = \mathbf{Q}_T$$

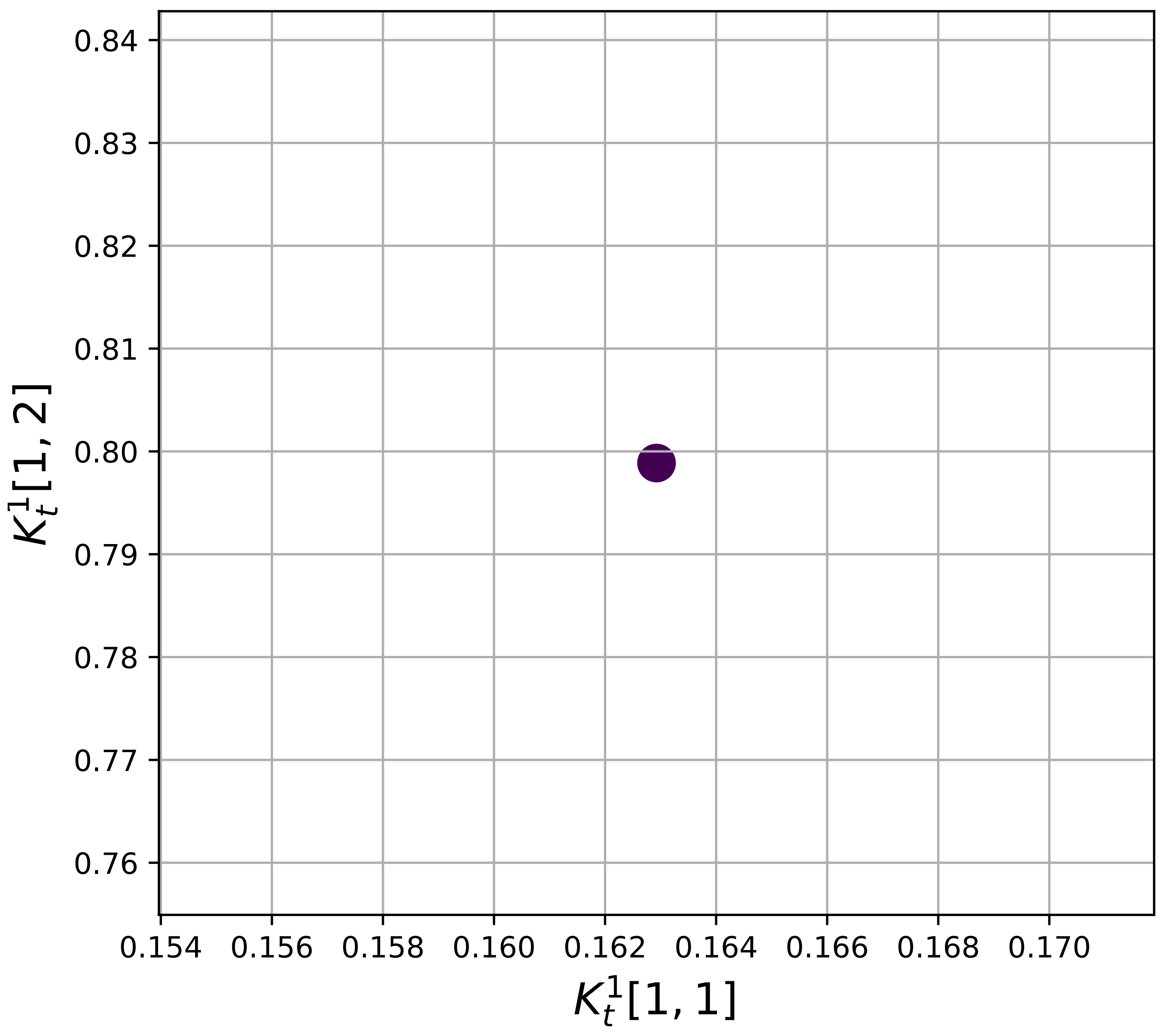
Multi agent - Nash equilibrium for the infinite-horizon LQR

$$\left(P^1, P^2, \ldots, P^N\right) = f\left(P^1, P^2, \ldots, P^N\right) \longrightarrow \mathbf{P} = f\left(\mathbf{P}\right)$$

$$\left(K^1, K^2, \ldots, K^N\right) = g\left(P^1, P^2, \ldots, P^N\right)$$

Evolution of the dynamical system $P_t = f\left(P_{t+1}\right)$

Fixed point



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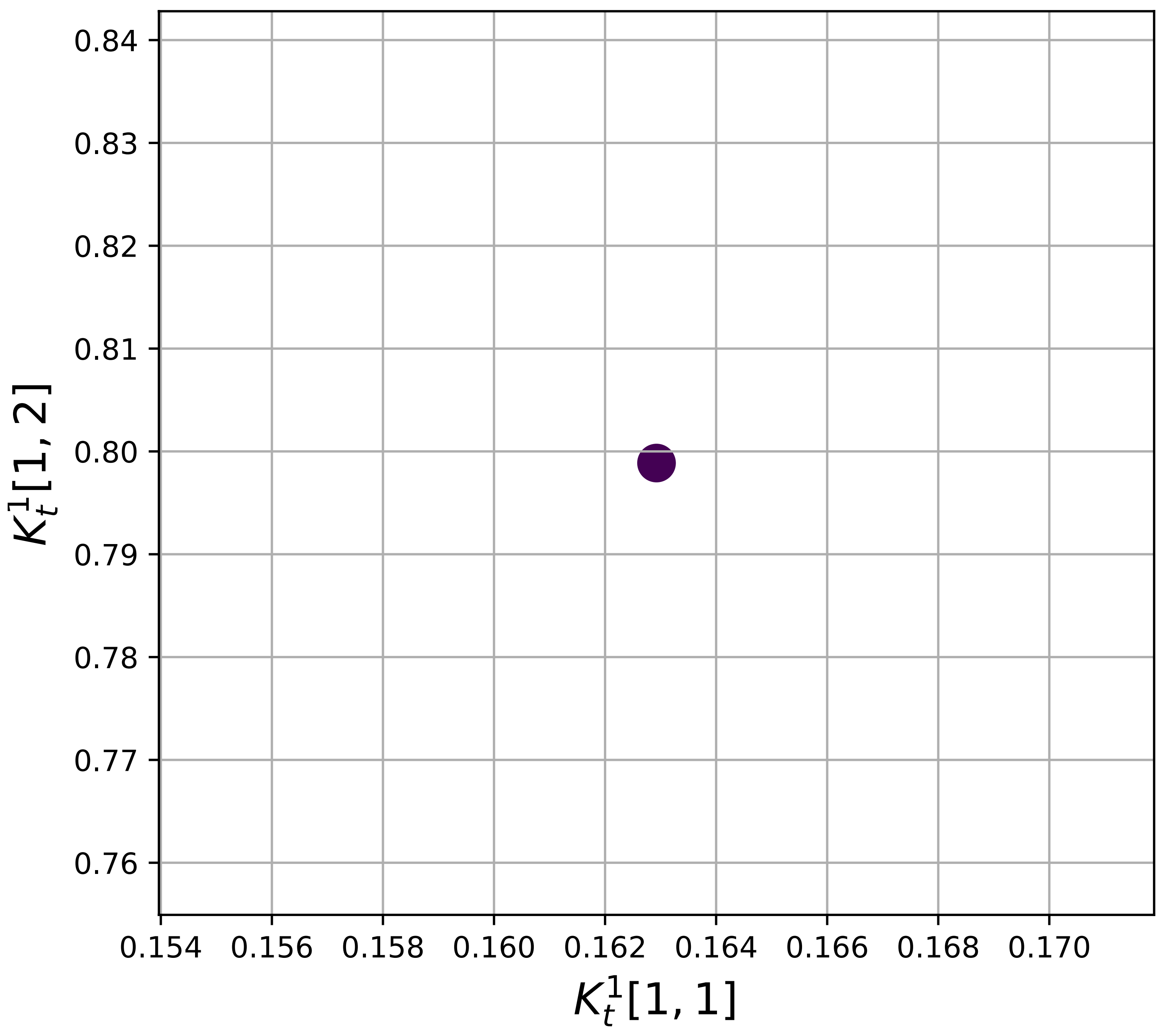
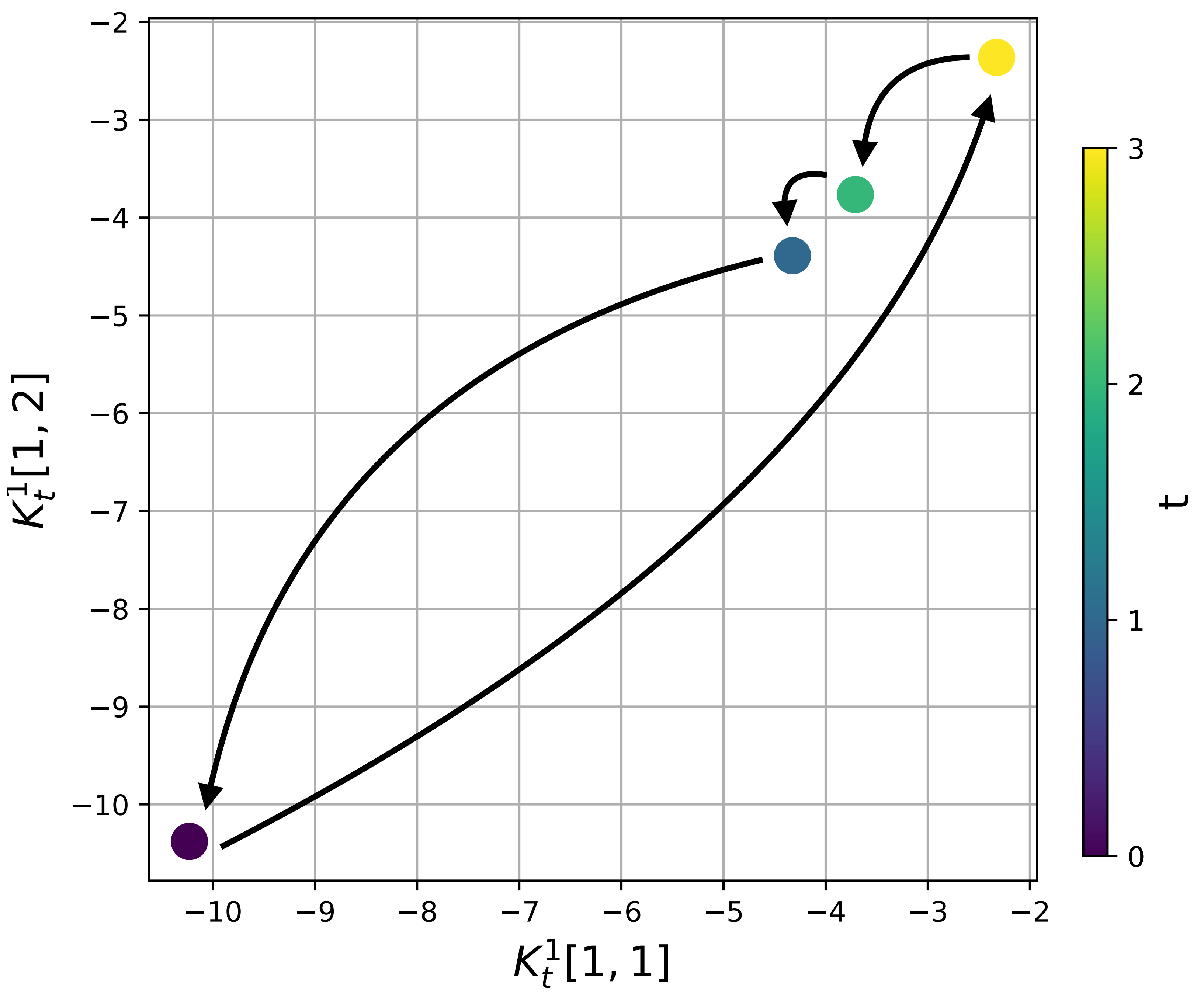


Periodic orbit



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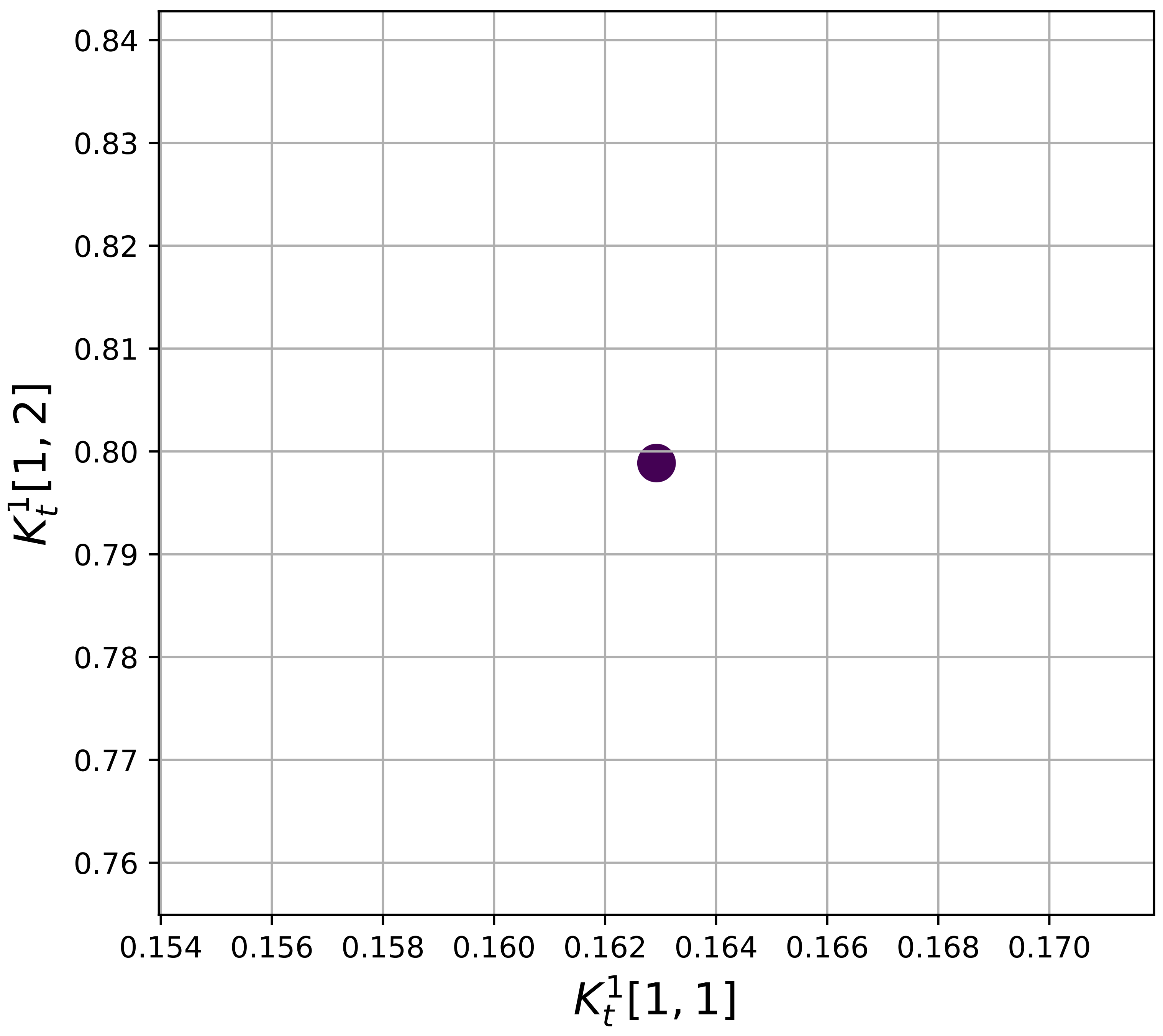

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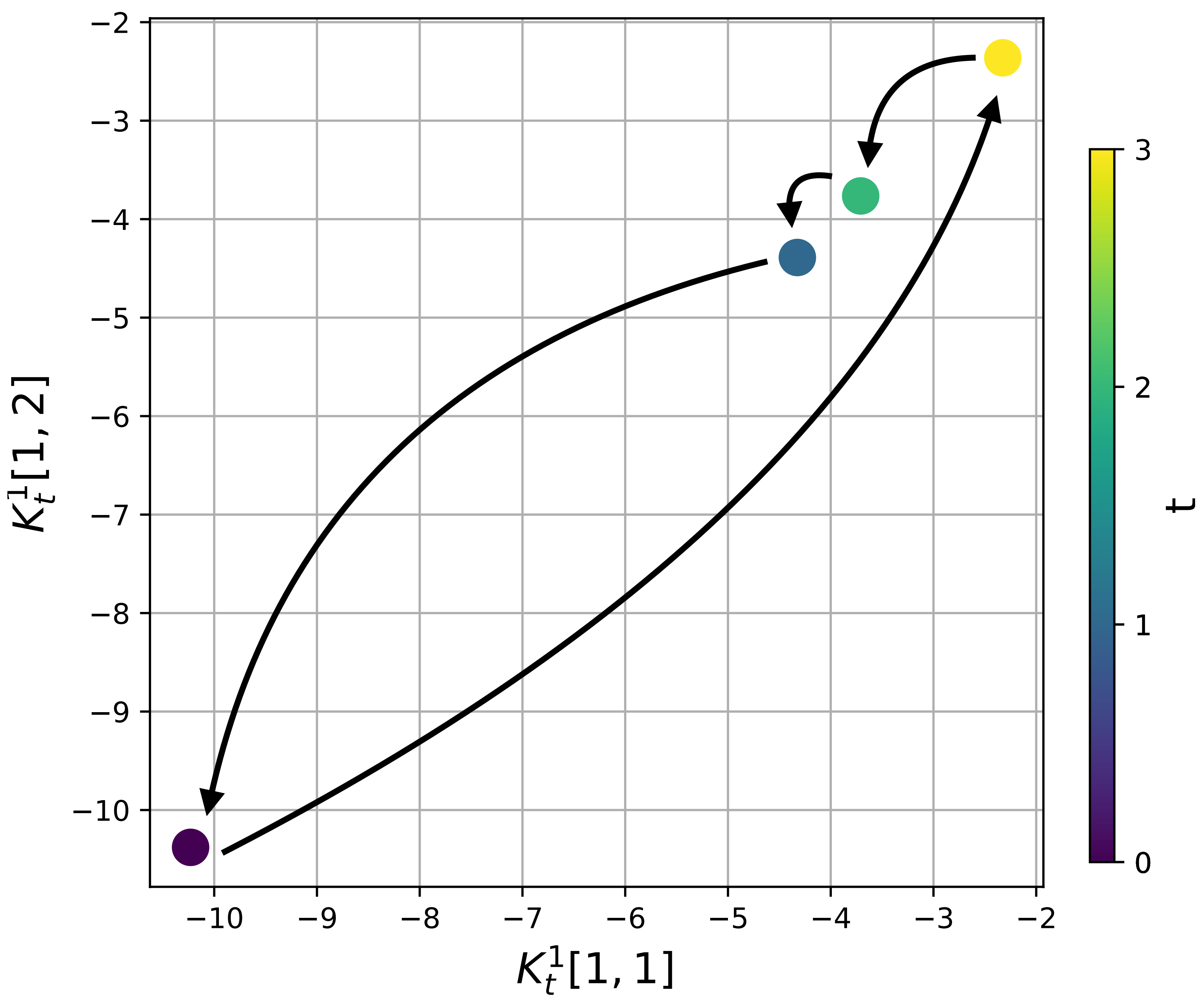
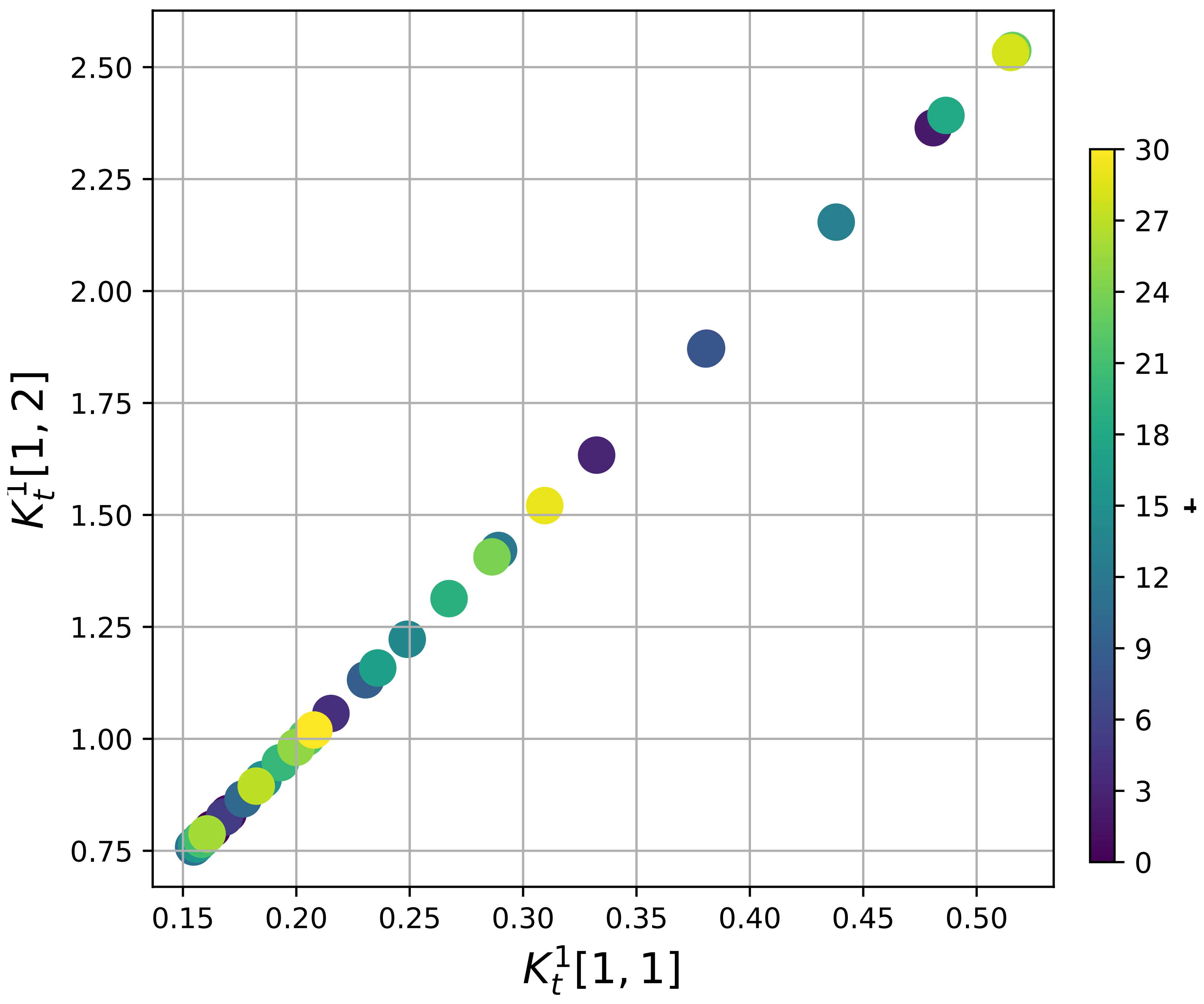


Bounded non-convergent



Periodic Nash equilibria

A set of matrices $\{\mathbf{P}_l\}_{l=1}^{L}$ constitutes a cycle if

$$\mathbf{P}_l = f(\mathbf{P}_{l+1}), \quad l \in [1, L-1], \quad \mathbf{P}_L = f(\mathbf{P}_1).$$

Theorem: Consider a cycle $\{\mathbf{P}_l\}_{l=1}^{L}$ for a stabilizable game with $Q^i \succ 0$ for all $i \in [1,N]$. Then $\{\mathbf{K}_l\}_{l=1}^{L}$, with $\mathbf{K}_l = g\left(\mathbf{P}_l\right)$, constitutes a Nash equilibrium.

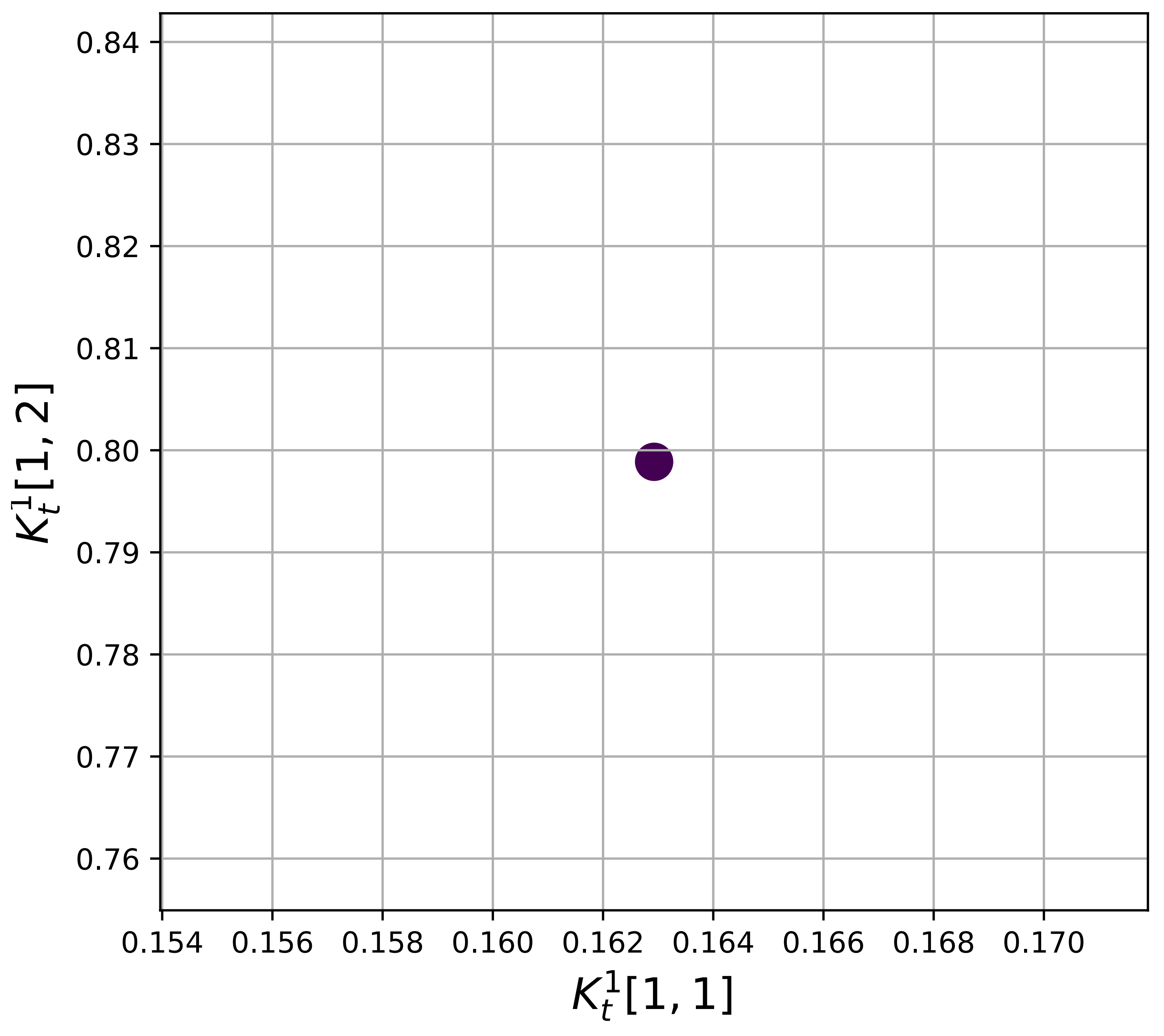
Main results

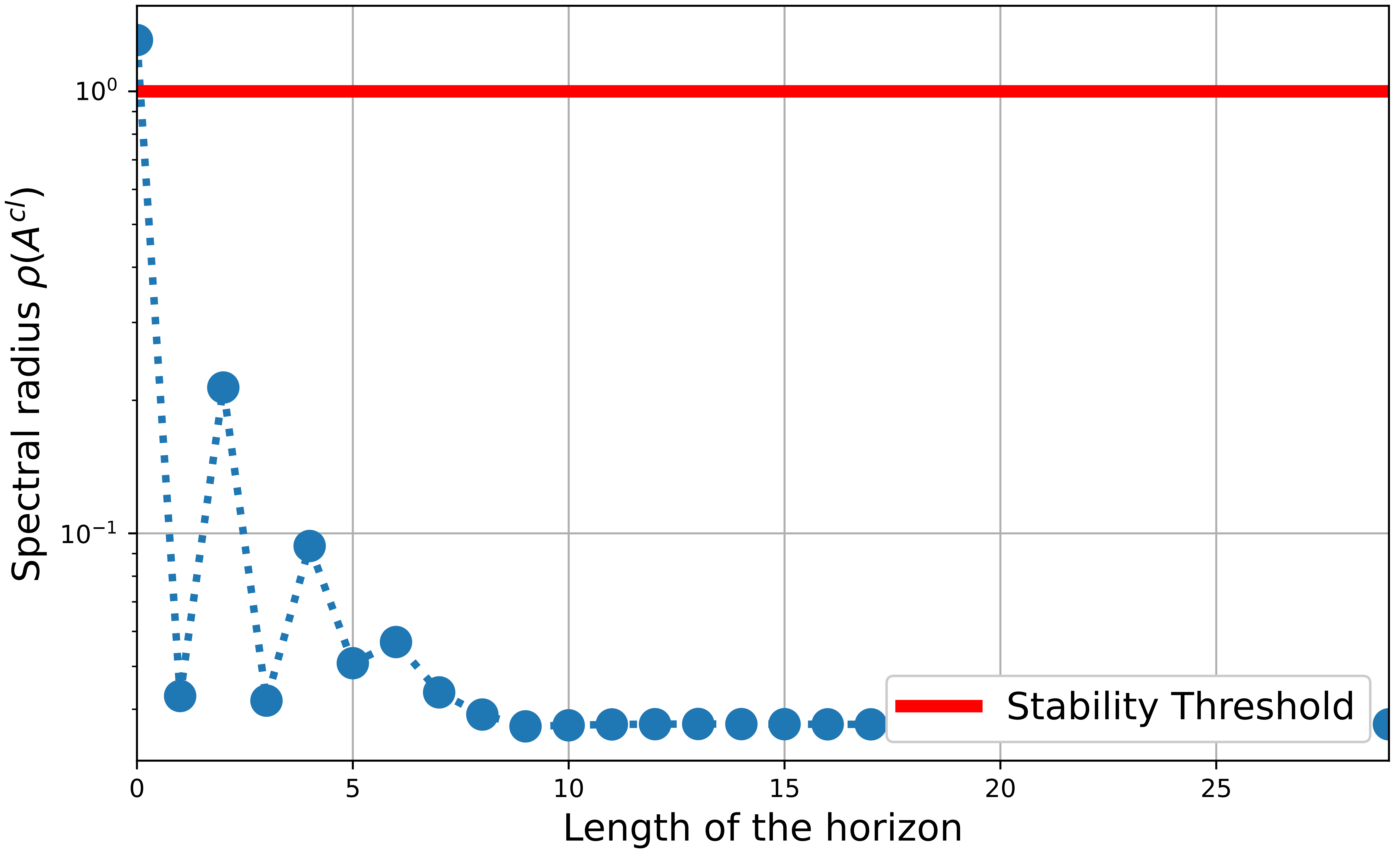
1. Extending the horizon is not enough to guarantee stability.

2. Terminal costs act as an equilibrium selector.

Stability of receding horizon games

Fixed point





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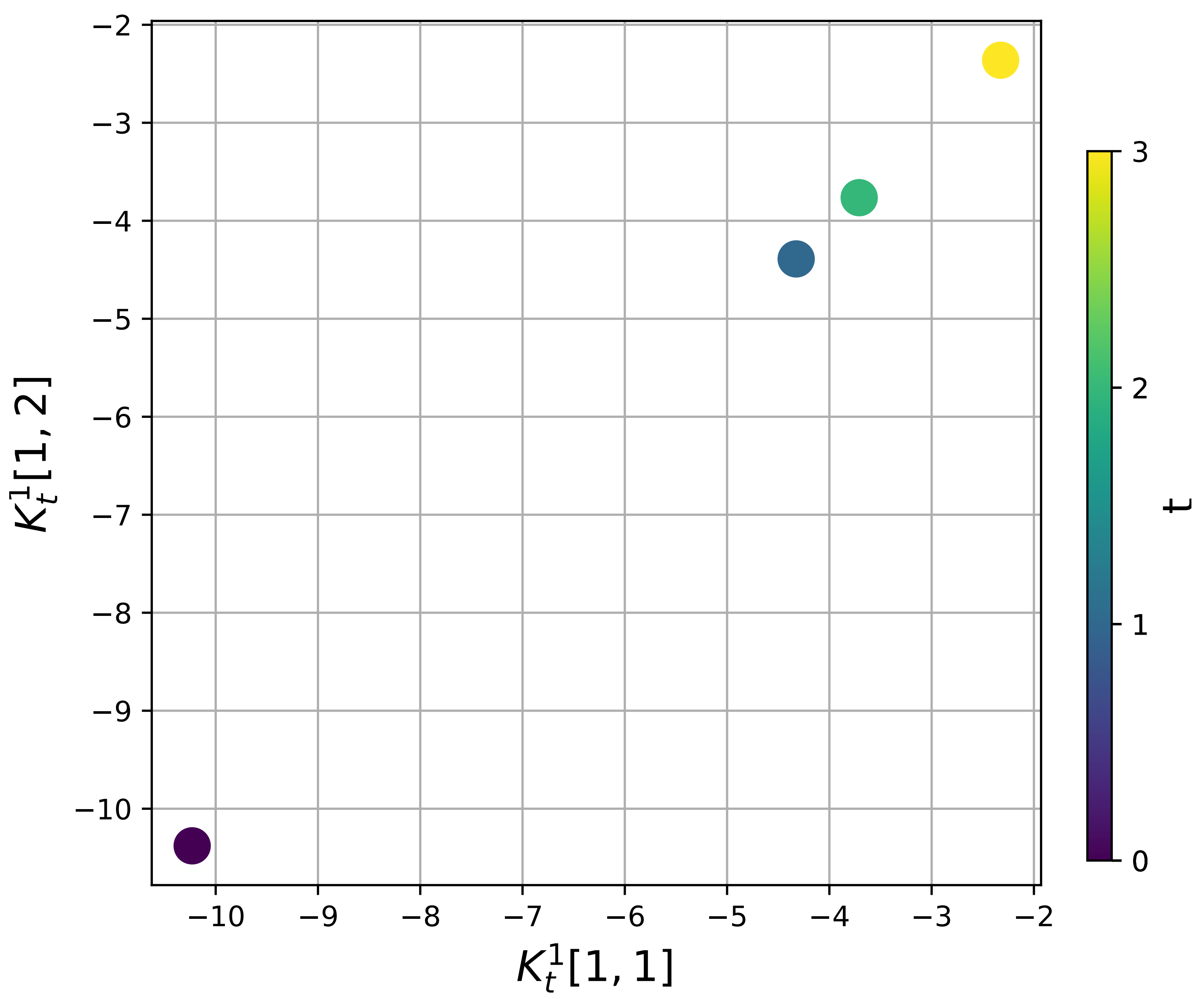

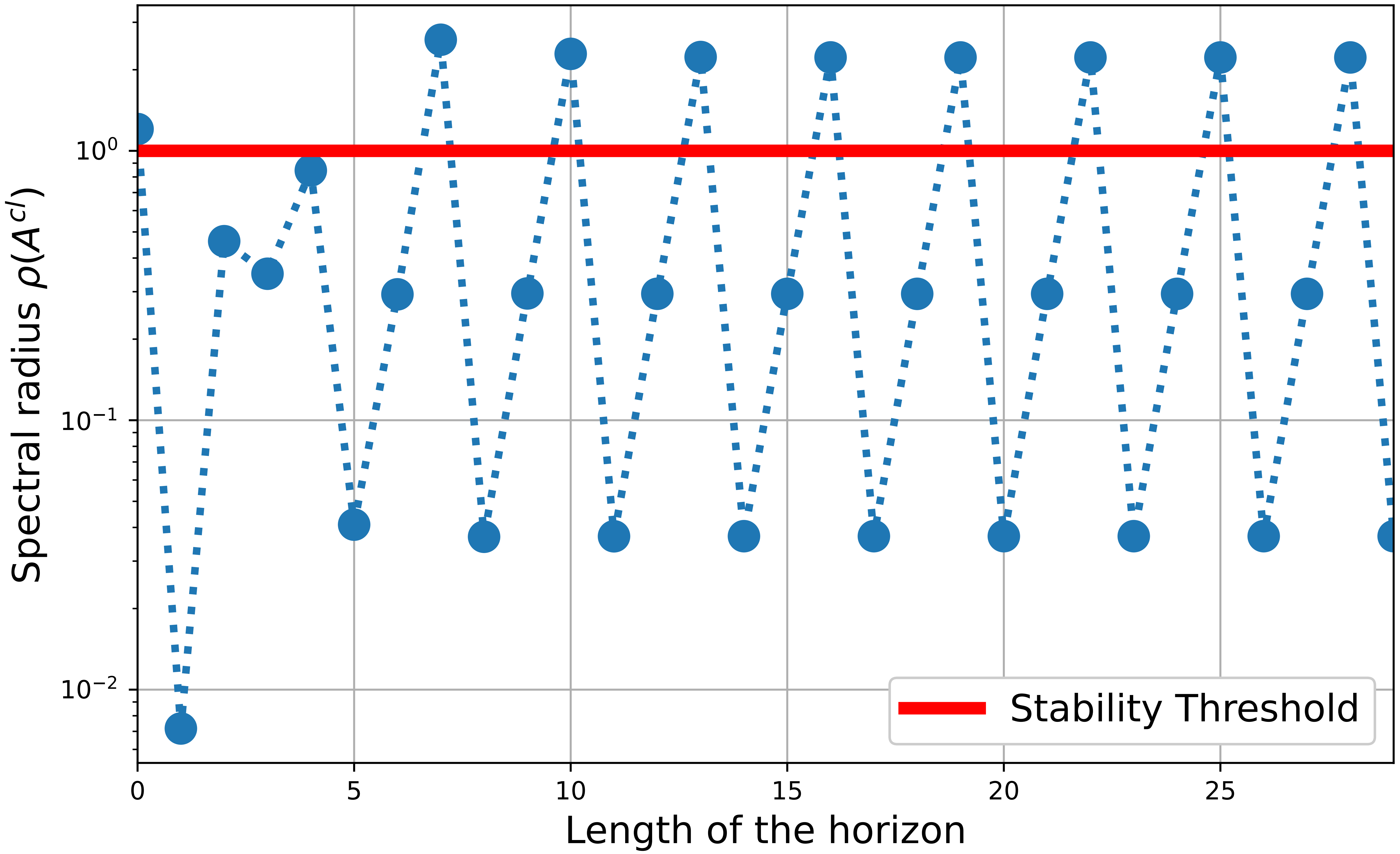

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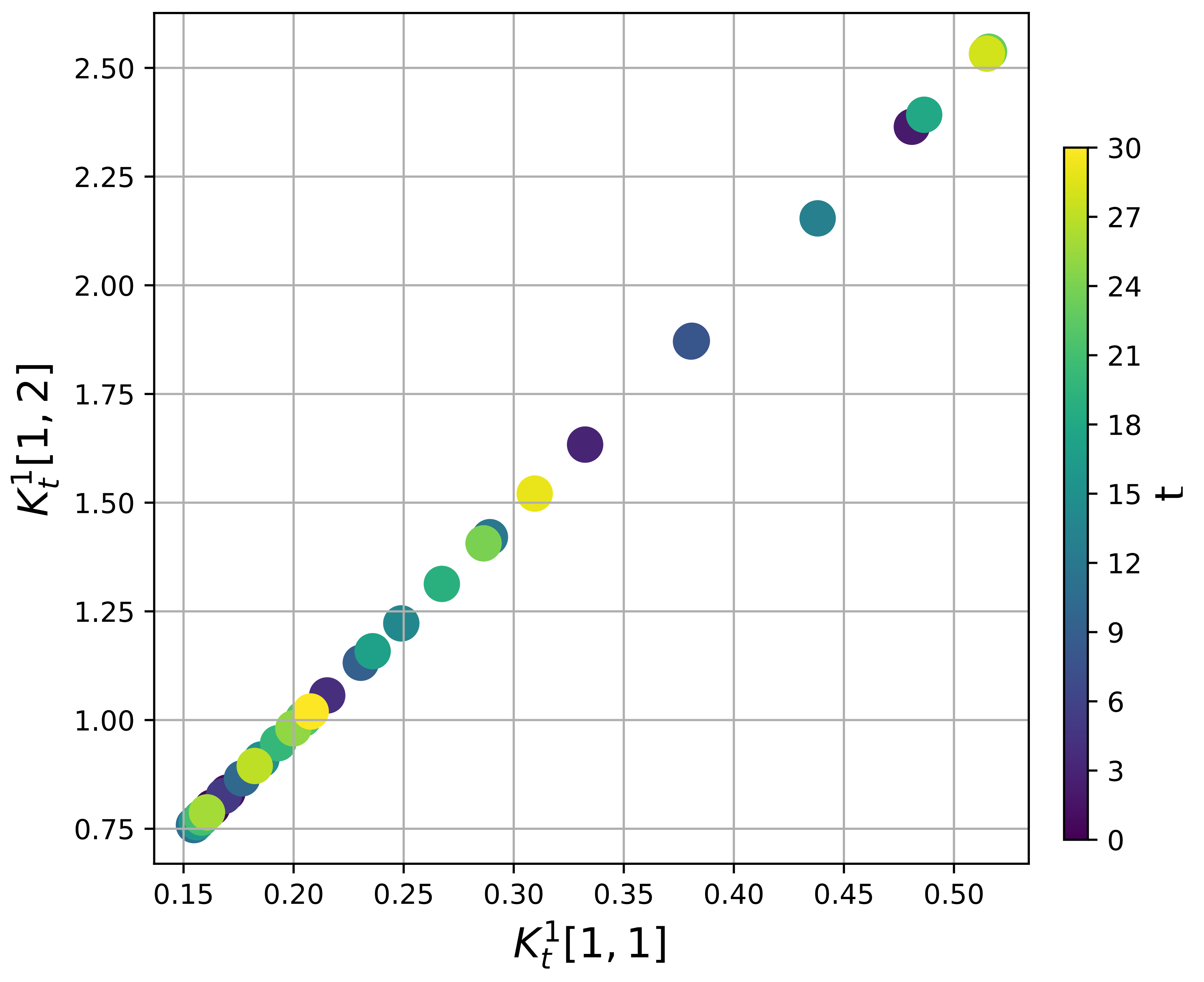

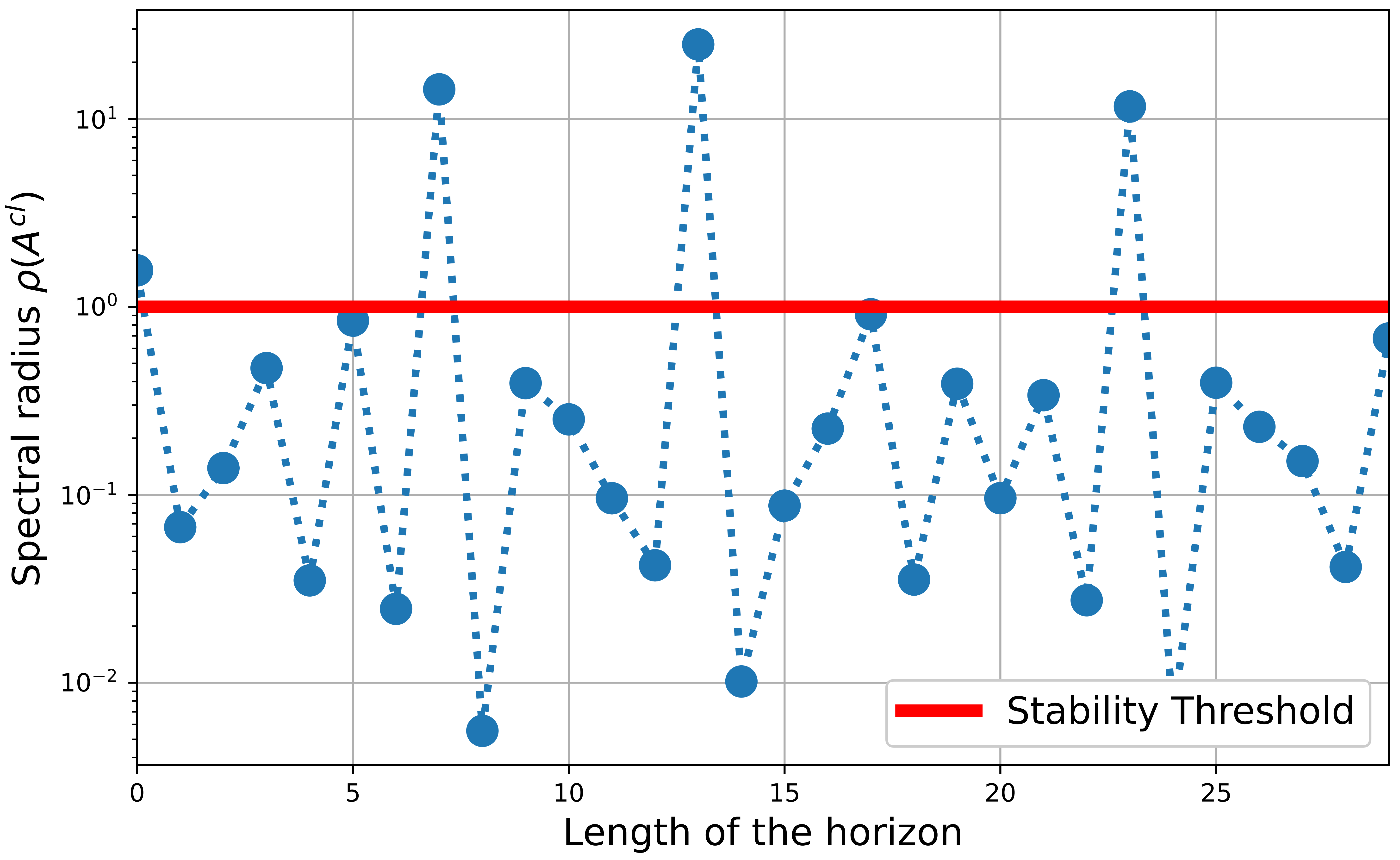

If the finite-horizon recursion does not converge to a fixed point, the length of the horizon does not matter.

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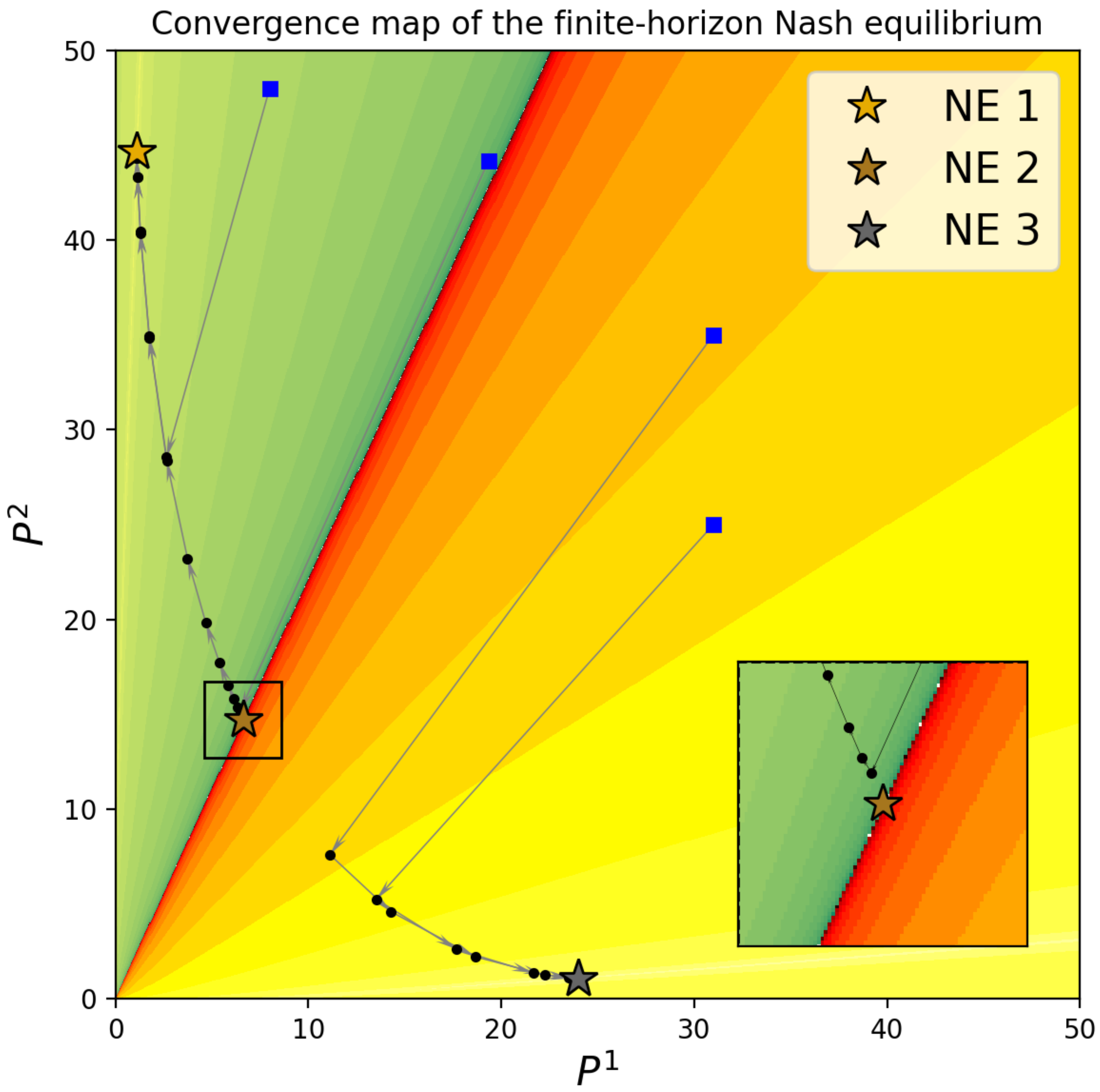
Attractive and saddle fixed points

Consider a scalar game with two agents and the following setting:

$A = 3, B_1 = B_2 = 1,$

$Q_1 = Q_2 = R_1 = 1, R_2 = 2.$



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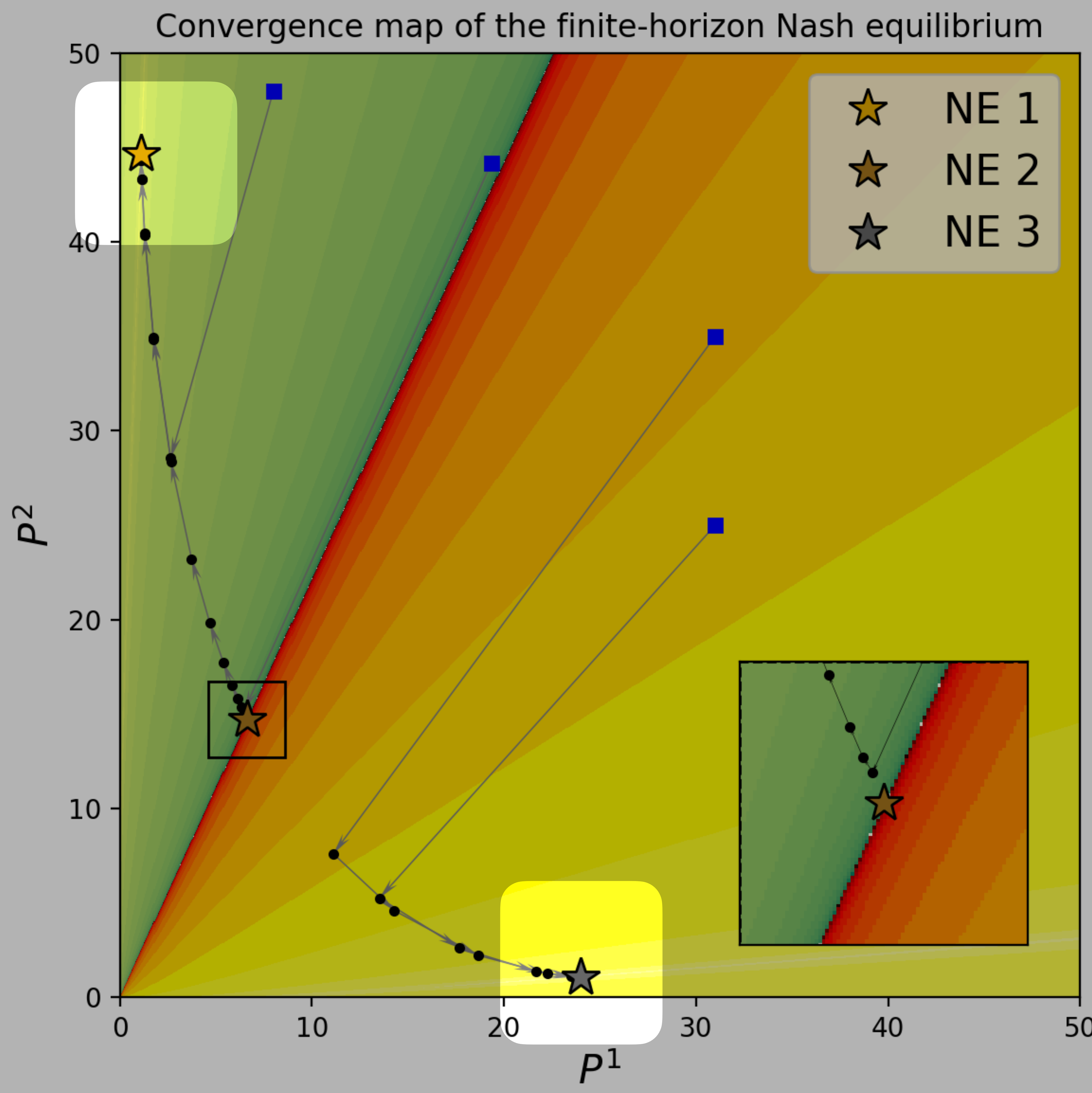

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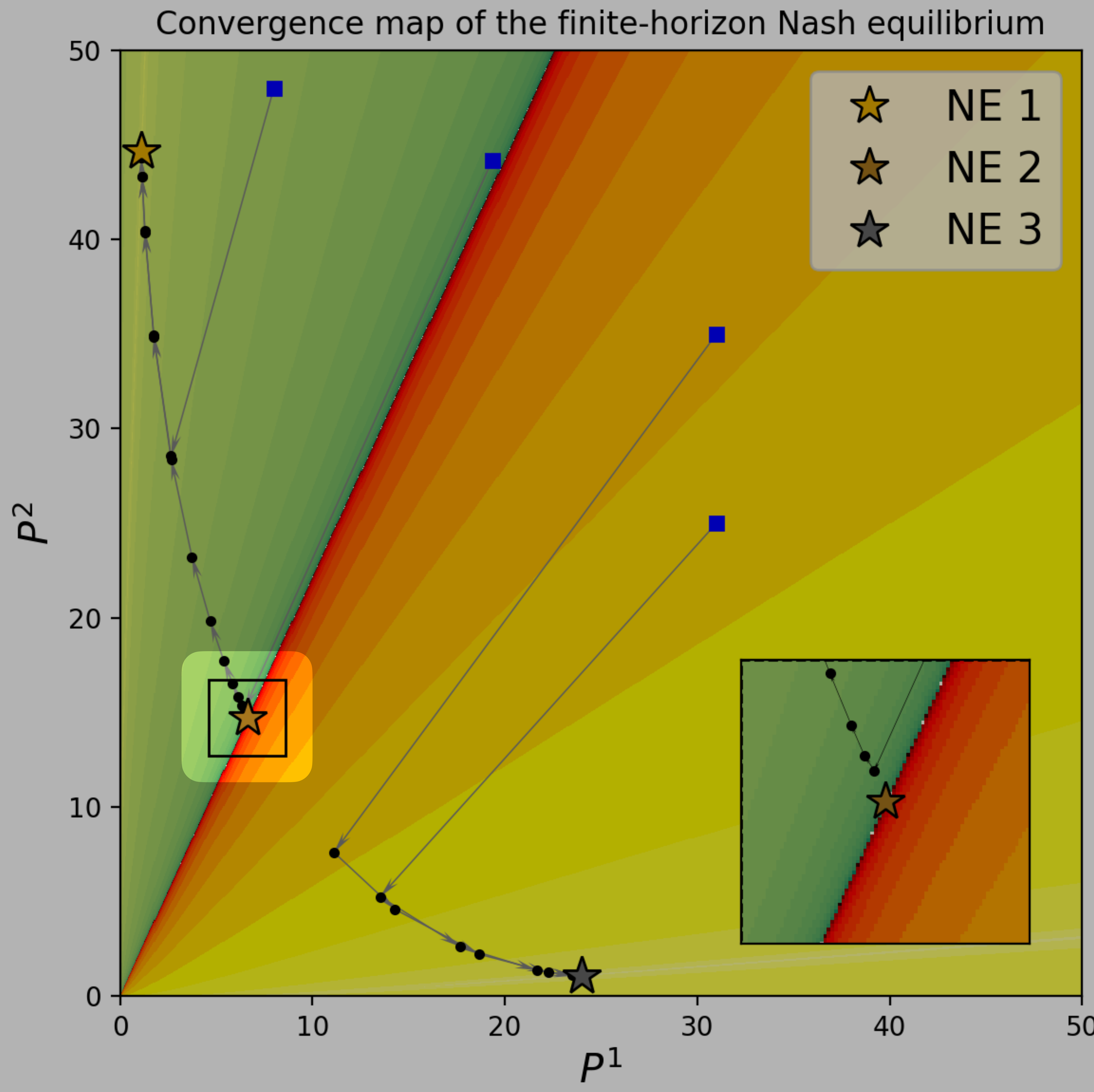

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Not all the Nash equilibria can be approximated. Only the ones attractive for the finite-horizon recursion.

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Open problems and future directions

1. How would constraints affect the results?
2. Can we reformulate the finite-horizon cost to guarantee convergence?
3. How can we compute all the Nash equilibria?